

Soft pion theorem for hard near threshold pion production

P.V. Pobylitsa^{a,b}, M.V. Polyakov^{a,b}, M. Strikman^{a,c}

^aPetersburg Nuclear Physics Institute, 188350, Gatchina, Russia

^b Institut für Theoretische Physik II, Ruhr-Universität Bochum,
D-44780 Bochum, Germany

^c Pennsylvania State University, University Park, PA 16802, USA

Abstract

We prove a new soft pion theorem for the near threshold pion production by a hard electromagnetic probe. This theorem relates various near threshold pion production amplitudes to the nucleon distribution amplitudes. The new soft pion theorem is in a good agreement with the SLAC data for the structure function $F_2^p(W, Q^2)$ for $W^2 \leq 1.4 \text{ GeV}^2$ and $7 \leq Q^2 \leq 30.7 \text{ GeV}^2$.

Introduction

The amplitudes of the pion production near the threshold by the electromagnetic probe

$$\gamma^* + N \rightarrow \pi + N' \quad (1)$$

at a not too large virtuality (Q^2) of the photon $Q^2 \ll \Lambda^3/m_\pi$ ($\Lambda \sim 1 \text{ GeV}$ is a typical hadronic scale) can be expressed in terms of various nucleon form factors with the help of the soft pion theorem (SPT) [1, 2, 3]. For virtualities $Q^2 \sim \Lambda^3/m_\pi$ and larger this SPT does not work [1, 2, 3].

In the present paper we derive a new “hard-soft” pion theorem (hSPT) for the reaction (1) in the near threshold region and for large virtuality of the photon $Q^2 \gg \Lambda^2$. Our main tool is the QCD factorization theorem for exclusive processes [4, 5] (for a recent review and comprehensive list of references see e.g. [6]). It allows us to express the pion production amplitude at large virtuality in terms of the distribution amplitudes (DAs) of the nucleon and of the low-mass πN system. These non-perturbative objects correspond to the lowest three quark (3q) Fock component of the nucleon and πN systems. We derive a hSPT to relate the corresponding distribution amplitude of the πN system (we shall call it as πN DA) to the nucleon distribution amplitude. This hSPT is valid for the limit when the mass of the πN system (denoted as W) is close to the threshold $W_{\text{th}} = M_N + m_\pi$, *i.e.* $W - W_{\text{th}} \lesssim m_\pi$. The derivation of the similar theorem for DA of the two pion system near threshold can be found in [7, 8].

The physical picture of the near threshold production of pion by a hard electromagnetic probe is as follows. At large Q^2 the emission of the soft pion from the initial state contributes only to large invariant masses W . The emission from the hard interaction part is a higher twist in Q^2 . Hence the emission occurs solely in the final state when a small $3q$ system produced in the hard interaction expands to a large enough configuration. At this point we are dealing with a soft pion emission and can apply corresponding near threshold chiral theory relations.

Threshold theorem for hard electromagnetic probe

We consider the hard reaction (1) in the near threshold region, *i.e.* when W is close to the threshold of the reaction $W_{\text{th}} = M_N + m_\pi$ and at large virtuality of the photon $Q \gg \Lambda$. Hence, $x_{Bj} \rightarrow [1 + m_\pi(2M_N + m_\pi)/Q^2]^{-1}$ in the discussed limit. In the near threshold region the cross section of the reaction (1) can be expressed in terms of the $N \rightarrow \pi N$ transition form factors at large momentum transfer Q^2 :

$$M_\mu^{ff'a} = \langle \pi^a N_{f'} | J_\mu^{\text{e.m.}}(0) | N_f \rangle \quad (2)$$

Here $f, f' = p, n$ are flavours of the initial and final nucleons, a is the flavour index of the emitted pion.

In this section we shall be interested in the matrix element (2) at the threshold of the reaction (1), *i.e.* at $W = W_{\text{th}}$. The matrix element (2) at the threshold for the transverse photon has the form*:

$$M_{\mu\perp}^{ff'a} = A(\gamma^* N_f \rightarrow N_{f'} + \pi^a) \bar{u}(p') \gamma_{\mu\perp} \gamma_5 u(p). \quad (3)$$

It follows from the QCD factorization theorem [4, 5] that the transition matrix element $A(\gamma^* N \rightarrow \pi N')$ at large Q^2 can be written as (up to the power suppressed terms)

$$A(\gamma^* N \rightarrow \pi N') = \int dx dy \Phi_{\pi N'}^*(x) T(x, y, Q^2) \Phi_N(y), \quad (4)$$

where $T(x, y, Q^2)$ is the hard part of the process computable in perturbative QCD. The functions $\Phi_N(y)$, $\Phi_{\pi N'}(x)$ are distribution amplitudes (light cone wave functions) of the nucleon and of the low-mass $\pi N'$ system. The DA of the nucleon $\Phi_N(y)$ also enters the QCD description of the nucleon form factor and it is a subject of intensive studies. The distribution amplitude of the πN system is a straightforward generalization of the baryon DAs. Many general properties of these objects are similar to those of the two-pion distribution amplitude which were extensively studied in Ref. [8]. They will be discussed elsewhere.

We focus here on the soft pion theorem for πN DAs. Using the general soft pion theorem (see e.g. [9]) we can write:

$$\begin{aligned} \langle \pi^a(k) N_f(p, S) | O | 0 \rangle &= \frac{i}{f_\pi} \langle N_f(p, S) | [Q_5^a, O] | 0 \rangle \\ &+ \frac{ig_A}{4f_\pi(p \cdot k)} \sum_{S', f'} \bar{u}(p, S) \not{k} \gamma_5 \tau_{ff'}^a u(p, S') \langle N_{f'}(p, S') | O | 0 \rangle. \end{aligned} \quad (5)$$

Here $f_\pi = 93$ MeV is the pion decay constant, $g_A \approx 1.25$ is the axial charge of the nucleon. The operator O is the trilocal quark operator of twist-3 in which quark fields are separated by a light cone distance, in the complete analogy with the definition of the baryon DA, see e.g. [10]. The operator Q_5^a is the operator of axial charge, so that the calculation of the commutator $[Q_5^a, O]$ reduces to the chiral rotation of operator O . The second term in the rhs of Eq. (5) corresponds to the chiral singularity due to the nucleon pole in the graph shown

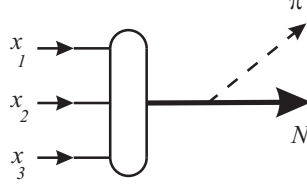


Figure 1: Nucleon pole contribution to the soft pion theorem for generalized πN distribution amplitudes.

in Fig. 1. The contribution of this diagram is strongly suppressed for $W - W_{\text{th}} \ll m_\pi$ but for $W - W_{\text{th}} \sim m_\pi$ it becomes significant, see Eqs.(24,25).

Let us start from the calculation of the first (commutator) term in Eq. (5). Since the chiral rotation of the trilocal quark operator O does not change its twist Eq. (5) allows us to express generalized πN DAs at the threshold in terms of nucleon DAs.

We write the nucleon DA in terms of functions $\phi_S(x)$ and $\phi_A(x)$ which are symmetric and antisymmetric respectively under $x_1 \leftrightarrow x_3$ (1 and 3 are quarks with parallel helicities) [4, 11]. For the proton we have

$$\begin{aligned}
 |p \uparrow\rangle &= \frac{\phi_S(x)}{\sqrt{6}} |2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle \\
 &+ \frac{\phi_A(x)}{\sqrt{2}} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle.
 \end{aligned} \tag{6}$$

The distribution amplitude for neutron can be obtained from the above expression by the replacement $u \leftrightarrow d$.

Applying the general soft pion theorem (5) we express the distribution amplitudes of various πN systems at the threshold in terms of the nucleon DAs $\phi_S(x)$ and $\phi_A(x)$:

$$\begin{aligned}
 |p \uparrow \pi^0\rangle &= \frac{\phi_S(x)}{2\sqrt{6}f_\pi} |6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle \\
 &- \frac{\phi_A(x)}{2\sqrt{2}f_\pi} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle,
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 |n \uparrow \pi^+\rangle &= \frac{\phi_S(x)}{\sqrt{12}f_\pi} |2u_\uparrow d_\downarrow u_\uparrow - 3 u_\uparrow u_\downarrow d_\uparrow - 3 d_\uparrow u_\downarrow u_\uparrow\rangle \\
 &- \frac{\phi_A(x)}{2f_\pi} |u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle.
 \end{aligned} \tag{8}$$

The DAs of the neutral πN systems can be obtained by the trivial replacement $u \leftrightarrow d$.

Now we can compute the threshold amplitudes $A(\gamma^* N \rightarrow \pi N')$ at large Q^2 combining the factorization theorem (4) with the expressions for πN DAs (7,8). The technique of calculations of the hard part $T(x, y, Q^2)$ is standard and can be found, e.g. in refs. [4, 10].

*Longitudinally polarized photon gives a power suppressed contribution for $Q^2 \gg \Lambda^2$.

We give here only the final results using notations of refs. [4, 11, 12]. For the transitions $p \rightarrow \pi N$ we obtain:

$$\begin{aligned}
Q^4 A(\gamma^* p \rightarrow \pi^0 p)|_{\text{th}} &= \frac{(16\pi\alpha_s)^2}{9f_\pi} \int [dx \, dy] \left\{ \frac{1}{9} (23T_1 - 8T_2) \phi_S(x) \phi_S(y) \right. \\
&+ \frac{1}{\sqrt{3}} T_1 [\phi_S(x) \phi_A(y) + \phi_S(y) \phi_A(x)] \\
&- \left. \frac{1}{3} (T_1 + 2T_2) \phi_A(y) \phi_A(x) \right\}, \tag{9}
\end{aligned}$$

$$\begin{aligned}
Q^4 A(\gamma^* p \rightarrow \pi^+ n)|_{\text{th}} &= \frac{(16\pi\alpha_s)^2}{9\sqrt{2}f_\pi} \int [dx \, dy] \left\{ \frac{2}{9} (11T_1 + 4T_2) \phi_S(x) \phi_S(y) \right. \\
&- \frac{2}{\sqrt{3}} T_1 [\phi_S(x) \phi_A(y) + \phi_S(y) \phi_A(x)] \\
&- \left. \frac{2}{3} (T_1 + 2T_2) \phi_A(y) \phi_A(x) \right\}. \tag{10}
\end{aligned}$$

Here $[dx] = dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$ and coefficient functions T_1 and T_2 have the form [4]:

$$\begin{aligned}
T_1 &= \frac{1}{x_3(1-x_1)^2 y_3(1-y_1)^2} + \frac{1}{x_2(1-x_1)^2 y_2(1-y_1)^2} \\
&- \frac{1}{x_2 x_3 (1-x_3) y_2 y_3 (1-y_1)}, \tag{11}
\end{aligned}$$

$$T_2 = \frac{1}{x_1 x_3 (1-x_1) y_1 y_3 (1-y_3)}. \tag{12}$$

Expressions for various nucleon form factors in the same notations can be found e.g. in refs. [4, 11, 12].

For the transitions $n \rightarrow \pi N$ we obtain:

$$\begin{aligned}
Q^4 A(\gamma^* n \rightarrow \pi^0 n)|_{\text{th}} &= \frac{(16\pi\alpha_s)^2}{9f_\pi} \int [dx \, dy] \left\{ \frac{13}{9} (T_1 - T_2) \phi_S(x) \phi_S(y) \right. \\
&+ \frac{1}{\sqrt{3}} T_1 [\phi_S(x) \phi_A(y) + \phi_S(y) \phi_A(x)] \\
&+ \left. \frac{1}{3} (T_1 - T_2) \phi_A(y) \phi_A(x) \right\}, \tag{13}
\end{aligned}$$

$$\begin{aligned}
Q^4 A(\gamma^* n \rightarrow \pi^- p)|_{\text{th}} &= \frac{(16\pi\alpha_s)^2}{9\sqrt{2}f_\pi} \int [dx \, dy] \left\{ -\frac{2}{9} (T_1 - T_2) \phi_S(x) \phi_S(y) \right. \\
&+ \frac{2}{\sqrt{3}} T_1 [\phi_S(x) \phi_A(y) + \phi_S(y) \phi_A(x)] \\
&- \left. \frac{2}{3} (T_1 - T_2) \phi_A(y) \phi_A(x) \right\}. \tag{14}
\end{aligned}$$

Using Eq. (9) one can express the near threshold structure functions $F_2^{p,n}(W, Q^2)$ and various differential cross sections for particular πN channels in terms of the nucleon DAs. The *same* DAs appear in the QCD factorization theorem for the nucleon form factors at large Q^2 . We find that in the case of the symmetric form of the nucleon DA

$$\phi_S(x) \text{ is arbitrary, } \phi_A(x) = 0, \quad (15)$$

one can describe the near threshold pion production directly in terms of the nucleon form factors without specifying the nucleon DA, see below. Certainly the nucleon DA can have a nonzero asymmetric component $\phi_A(x)$. Actually our general Eq. (9) can be applied to any specific model of the nucleon DA (see e.g. [13]) to compare the model predictions with the experiment. In the case of symmetric nucleon DA the amplitude of the process (1) can be expressed in terms of nucleon magnetic form factors ($G_{MN}(Q^2)$) as follows:

$$A(\gamma^* p \rightarrow \pi^0 p)|_{\text{th}} = -\frac{1}{f_\pi} \left(\frac{5}{6} G_{Mp} - \frac{4}{3} G_{Mn} \right), \quad (16)$$

$$A(\gamma^* p \rightarrow \pi^+ n)|_{\text{th}} = \frac{1}{\sqrt{2}f_\pi} \left(\frac{5}{3} G_{Mp} + \frac{4}{3} G_{Mn} \right), \quad (17)$$

$$A(\gamma^* n \rightarrow \pi^0 n)|_{\text{th}} = -\frac{13}{6f_\pi} G_{Mn}, \quad (18)$$

$$A(\gamma^* n \rightarrow \pi^- p)|_{\text{th}} = \frac{G_{Mn}}{3\sqrt{2}f_\pi}. \quad (19)$$

Since these results are based on the symmetric form (15) for the nucleon distribution amplitude, deviations from these equations would allow to probe directly the asymmetric part of the nucleon distribution amplitude and check the validity of the leading twist description of the nucleon form factors.

At asymptotically large Q^2 the DAs of the nucleon are evolved to their asymptotic values:

$$\phi_S^{\text{as}}(x) = N x_1 x_2 x_3, \quad \phi_A^{\text{as}}(x) = 0. \quad (20)$$

As the form (up to overall normalization) of the asymptotic nucleon DAs is fixed uniquely we can make predictions for the ratios of various amplitudes at the threshold at $Q^2 \rightarrow \infty$ $\lim_{Q^2 \rightarrow \infty} \frac{A(\gamma^* p \rightarrow \pi^0 p)|_{\text{th}}}{A(\gamma^* p \rightarrow \pi^+ n)|_{\text{th}}} = -\sqrt{2} + O\left(\frac{m_\pi}{M_N}\right)$, $\lim_{Q^2 \rightarrow \infty} \frac{A(\gamma^* n \rightarrow \pi^0 n)|_{\text{th}}}{A(\gamma^* n \rightarrow \pi^- p)|_{\text{th}}} = -\frac{13}{\sqrt{2}} + O\left(\frac{m_\pi}{M_N}\right)$ which are qualitatively different from predictions for the case $Q^2 \rightarrow 0$ [1] $\lim_{Q^2 \rightarrow 0} \frac{A(\gamma^* N \rightarrow \pi^0 N)|_{\text{th}}}{A(\gamma^* p \rightarrow \pi^+ n)|_{\text{th}}} = O\left(\frac{m_\pi}{M_N}\right)$ For the ratio of neutron to proton structure functions we obtain from new soft pion theorems the following asymptotic result: $\lim_{Q^2 \rightarrow \infty, W \rightarrow W_{\text{th}}} \frac{F_2^n(W, Q^2)}{F_2^p(W, Q^2)} = \frac{57}{32} + O\left(\frac{m_\pi}{M_N}\right)$, which is much larger than the perturbative QCD scaling limit expectation of 3/7 [14]. Although it is worth noting that the presently experimentally accessible values of Q^2 are not very large. Therefore one may expect considerable deviations from the above asymptotic results.

Structure functions: comparison with data

The data on the structure function $F_2^p(W, Q^2)$ in the near threshold region were obtained in 1994 in the SLAC experiment E136 [15] for a wide range of $Q^2 = 7 \div 30.7 \text{ GeV}^2$. Here we make the first comparison of the data with the hSPT predictions. (Previous analyses addressed the scaling features of these data and did not attempt to calculate $F_2^p(W \leq 1.2 \text{ GeV}, Q^2)$.)

To compute $F_2^p(W, Q^2)$ for $W - W_{\text{th}} \lesssim m_\pi$ we combine the strictly threshold amplitude (9) with the contribution of the last term from Eq. (5) and obtain

$$F_2^p(W, Q^2) = \frac{Q^2 \beta(W)}{(4\pi)^2} \left[\sum_{X=p\pi^0, n\pi^+} |A(\gamma^* p \rightarrow X)|_{\text{th}}^2 + \frac{3g_A^2 G_{Mp}^2(Q^2) \beta^2(W) W^4}{4(W^2 - M_N^2 + m_\pi^2)^2} + O\left(\frac{m_\pi}{\Lambda}\right) \right], \quad (21)$$

where

$$\beta(W) = \sqrt{1 - \frac{(M_N + m_\pi)^2}{W^2}} \sqrt{1 - \frac{(M_N - m_\pi)^2}{W^2}}. \quad (22)$$

The first term in the rhs of eq. (21) corresponds to strictly threshold amplitude (9). The second term takes into account the emission of the pion from the outgoing nucleon (Fig. 1) with the amplitude given by the second term in the rhs of eq. (5). The latter contribution vanishes exactly at the threshold but gives a parametrically unsuppressed contribution for $W - W_{\text{th}} \sim m_\pi$. Note that the second term in Eq. (21) corresponds to πN system in P-wave, therefore it can be separated from the first (S-wave) term by considering the angular distributions in πN system.

The data of E136 experiment Ref. [15] are consistent with the factorized form

$$F_2^p(W, Q^2) = F(Q^2) G(W), \quad (23)$$

with $F(Q^2) \propto 1/Q^6$ for $Q^2 \geq 8 \text{ GeV}^2$ which is exactly the scaling behaviour following from Eq. (21). We also found that Eq. (21) provides a good description of the W -dependence of the E136 data for $W \leq 1.2 \text{ GeV}$ though the predicted W -dependence is somewhat different from the $G(W) \propto W^2 - W_{\text{th}}^2$ fit of [15]. However, the resolution of E136 is not sufficient to distinguish between the two forms. The hSPT (21) also predicts the scaling behaviour $\sim 1/Q^6$ which is confirmed by the E136 data [15].

Thus we reproduce several features of the data without using any specific nucleon wave functions. To make a first quantitative comparison of the soft pion theorem prediction for the absolute value of F_2^p we use the symmetric form for the nucleon distribution amplitude (15). Inserting the expressions (16) and (17) for the threshold amplitudes into Eq. (21) and using the analogous expression for the neutron structure function, we obtain the following hSPT for the structure functions in the case of the symmetric form (15) for the nucleon DA

$$F_2^p(W, Q^2) = \frac{Q^2 \beta(W)}{(4\pi f_\pi)^2} \left[\frac{25}{12} G_{Mp}^2(Q^2) + \frac{8}{3} G_{Mn}^2(Q^2) + \frac{3g_A^2 G_{Mp}^2(Q^2) \beta^2(W) W^4}{4(W^2 - M_N^2 + m_\pi^2)^2} + O\left(\frac{m_\pi}{\Lambda}\right) \right], \quad (24)$$

$$F_2^n(W, Q^2) = \frac{Q^2 \beta(W)}{(4\pi f_\pi)^2} \left[\frac{19}{4} G_{Mn}^2(Q^2) + \frac{3g_A^2 G_{Mn}^2(Q^2) \beta^2(W) W^4}{4(W^2 - M_N^2 + m_\pi^2)^2} + O\left(\frac{m_\pi}{\Lambda}\right) \right]. \quad (25)$$

Now we can compare our results based on the hSPT with the E136 data [15]. It was found in this experiment that in the interval $W^2 \leq 1.4 \text{ GeV}^2$ the quantity $\int_{\text{th}}^{1.4} dW^2 Q^6 F_2^p(W, Q^2)$ is practically constant for $Q^2 \geq 8 \text{ GeV}^2$. The experimental value of the integral is in a good agreement with our result (24):

$$\int_{\text{th}}^{1.4} dW^2 Q^6 F_2^p(W, Q^2) = \begin{cases} 0.10 \pm 0.02 \text{ GeV}^8 & (\text{E136}) \\ 0.11 \pm 0.02 \text{ GeV}^8 & (\text{hSPT}) \end{cases} \quad (26)$$

For the theoretical analysis we used as input the following values for the nucleon form factors $Q^4 G_{Mp}(Q^2) = 1.0 \pm 0.1 \text{ GeV}^4$ obtained at $Q^2 \geq 10 \text{ GeV}^2$ [16] and $Q^4 G_{Mn}(Q^2) = -(0.5 \pm 0.1) \text{ GeV}^4$ extracted from Ref. [17] at $Q^2 \approx 10 \text{ GeV}^2$ [18]. For the summary of the current information on various baryon form factors at large momentum transfer see e.g. [19]. Note that the contribution of the P-wave term in Eq. (24) is relatively small (about 20%), so that the main part of the hSPT value (26) is due to the strictly threshold contribution (S-wave) in Eq. (21). With the same set of parameters we predict $\int_{\text{th}}^{1.4} dW^2 Q^6 F_2^n(W, Q^2) = 0.05 \pm 0.02 \text{ GeV}^8$. Note that for the nucleon DAs which fit $G_{Mp}(Q^2), G_{Mn}(Q^2)$ at $Q^2 \geq 10 \text{ GeV}^2$ (such DA significantly differ from the asymptotic one, see e.g. [11, 12]) the F_2^n/F_2^p ratio near threshold is much smaller than the asymptotic value of 52/37 which follows from Eq.(9) with the asymptotic distribution amplitude $\phi(x) \propto x_1 x_2 x_3$. Thus this ratio is extremely sensitive to the deviations of the nucleon DA from the asymptotic form. Therefore measurements of the neutron structure function in the near threshold region would considerably constrain the form of the nucleon distribution amplitude.

Conclusions

In this paper we derived a new soft pion theorem for the threshold pion production by a hard electromagnetic probe, i.e. with the probe of virtuality $Q^2 \gg \Lambda^2$ ($\Lambda \sim 1 \text{ GeV}$ is a typical hadronic scale). This new hSPT allows us to express the pion production amplitudes in terms of the distribution amplitudes of the nucleon. The latter enter the description of various nucleon form factors at large momentum transfer. These new relations give a possibility to constrain further the nucleon distribution amplitude using data on threshold inelastic electron scattering from the nucleon at high momentum transfer.

Here we restricted ourselves only to the case of the leading chiral contributions to the corresponding threshold amplitudes. The chiral corrections can be rather easily computed using methods of the chiral perturbation theory.

Using a generic symmetric model for the nucleon DAs we demonstrated that various observables for near threshold pion production at high momentum transfer are sensitive to the parameters of nucleon DAs. This shows that the near threshold pion production by a hard electromagnetic probe is a new valuable source of information about nucleon distribution amplitudes. Studies with a broader range of models of nucleon DAs will be presented elsewhere.

Our analysis was restricted to the leading twist QCD contributions. The application of the methods developed here to the models for soft contributions to the baryon form factors (see a review in [20, 21]) would allow one to derive predictions of these models for hard near threshold pion production. This might be an exciting possibility to use hSPT to discriminate between soft and hard mechanisms for high momentum transfer reactions.

We also note that the generalized πN distribution amplitudes above the threshold are complex functions. Not very far from the threshold the phases of πN DAs are fixed by known elastic πN scattering phase shifts which allows us to use the Omnès solution to the corresponding dispersion relations in order to fix the dependence of πN DAs on the mass of the corresponding πN system [8, 22]. Since this dependence is fixed to a large extent by the πN scattering phase shifts, the generalized πN DAs also contain information about DAs of various πN resonances. In a complete analogy with the two-pion distribution amplitudes the generalized πN DAs give a possibility to describe resonance and non-resonance contributions to the reaction (1) at high photon virtuality in a unified way.

The formalism developed here can be also used for other hard reactions with the pion emission near the threshold. For instance one can analyze the Compton scattering $\gamma + N \rightarrow \gamma + (\pi N)$ at large momentum transfer.

The study of the discussed processes should be feasible at the top of the current JLab energies and should be one of the high priorities of JLab at 12 GeV.

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References

- [1] N.M. Kroll, M.A. Ruderman, Phys. Rev. **93** (1954) 233;
Y. Nambu, D. Lurié, Phys. Rev. **125** (1962) 1429;
Y. Nambu, E. Shrauner, Phys. Rev. **128** (1962) 862.
- [2] A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. **B36** (1972) 589.
- [3] S. Scherer, J.H. Koch, Nucl. Phys. **A534** (1991) 461.
- [4] P. Lepage, S. Brodsky, Phys. Rev. Lett. **43** (1979) 545, Erratum-ibid. **43** (1979) 1625;
P. Lepage, S. Brodsky, Phys. Rev. **D22** (1980) 2157.
- [5] A. V. Efremov and A. V. Radyushkin, Theor. Math. Phys. **44**, 774 (1981).
- [6] S. J. Brodsky, “Exclusive processes in quantum chromodynamics and the light-cone Fock representation,” SLAC-PUB-8649, contribution to the Boris Ioffe Festschrift ‘At the Frontier of Particle Physics: A Handbook for QCD,’ edited by M. Shifman.
- [7] O. Teryaev, talk given at “XIV International Seminar on High Energy Physics Problems”, August 1998.
- [8] M. V. Polyakov, Nucl. Phys. **B555** (1999) 231.

- [9] S. Adler, R. Dashen, “Current algebras”, Benjamin, New York, 1968;
V. de. Alfaro, S. Fubini, G. Furlan, and C. Rossetti, “ Currents in hadron physics”, North-Holland, Amsterdam, 1973.
- [10] V. Chernyak, A. Zhitnitsky, Phys. Rep. **112** (1984) 173.
- [11] C.E. Carlson, J.L. Poor, Phys. Rev. **D34** (1986) 1478.
- [12] C.E. Carlson, M. Gari, N.G. Stefanis, Phys. Rev. Lett. **58** (1987) 1308.
- [13] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B246** (1984) 52;
M. Gari and N. G. Stefanis, Phys. Lett. **B175** (1986) 462; Phys. Rev. D **35** (1987) 1074;
V. L. Chernyak, A. A. Ogloblin and I. R. Zhitnitsky, Sov. J. Nucl. Phys. **48** (1988) 536;
N. G. Stefanis, Phys. Rev. D **40** (1989) 2305;
G. Martinelli and C. T. Sachrajda, Phys. Lett. **B217** (1989) 319;
N. Stefanis, M. Bergmann, Phys. Rev. **D47** (1993) 3685;
R. Jakob, P. Kroll, M. Schurmann, W. Schweiger, Z. Phys. **A347** (1993) 109;
J. Bolz and P. Kroll, Z. Phys. **A356** (1996) 327;
N. G. Stefanis, Eur. Phys. J. direct **C7** (1999) 1 [hep-ph/9911375].
- [14] G. R. Farrar and D. R. Jackson, Phys. Rev. Lett. **35** (1975) 1416.
- [15] P. Bosted *et al.*, Phys. Rev. D **49** (1994) 3091.
- [16] R. G. Arnold *et al.*, Phys. Rev. Lett. **57**, 174 (1986);
A. F. Sill *et al.*, Phys. Rev. **D 48** (1993) 29.
- [17] S. Rock *et al.*, Phys. Rev. **D 46** (1992) 24.
- [18] Note that in perturbative QCD $Q^4 G_{MN}(Q^2)$ should logarithmically decrease with Q^2 . The E134 data appear to support such a trend for $Q^2 > 10 \text{ GeV}^2$, see e.g. discussion in [6]. This would imply a similar decrease for the integral (26), for example by a factor ~ 1.5 between $Q^2=10$ and 31 GeV^2 . The data [15] do not contradict to such a decrease, though the errors become large for the data points at the highest Q^2 .
- [19] P. Stoler, Phys. Rep. **226** (1993) 103;
G. Sterman and P. Stoler, Ann. Rev. Nucl. Part. Sci. **43** (1997) 193.
- [20] A. V. Radyushkin, Few Body Syst. Suppl. **11**, 57 (1999) [hep-ph/9811225].
- [21] P. Kroll, Nucl. Phys. **A666** (2000) 3 [hep-ph/9907301].
- [22] M. V. Polyakov, Procs. of the 8th International Conference “Baryon’98”, Bonn, Sept. 1998, eqs. D.W. Menze, B. Metsch, World Scientific, pp. 765-769.